INSTITUTES

OF

PHYSICS.

VOLUME THIRD.

NON FINGENDUM AUT EXCOGITANDUM,
SED INVENIENDUM, QUID NATURA FACIAT
AUT FERAT. BACON.

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PART VI.

MECHANICS.

SECT. I.

GENERAL LAWS IN MECHANICS.

PROP. I.

Every body, or maſs of matter, continues in a ſtate of reſt, or of uniform motion in a ſtraight line, till it is made to change that ſtate by ſome new force impreſſed.

PROP. II.

The change of motion is always proportional to the moving force impreſſed, and is produced in the ſtraight line in which that force acts.

PROP. III.

Action and reaction are equal and contrary: that is, there is a reaction equal to every action of one body upon another, and in a contrary direction; ſo that, when a greater force acts contrary to a leſſer force, it will only produce a motion that is proportional to the difference of the two forces.

PROP. IV.

If a body is acted upon by two forces, the directions of which make an angle, the body will move in the diagonal of a parallelogram, the lengths of two contiguous ſides of which are in the ſame proportion to each other which the reſpective forces are: and it will go thro the diagonal in the ſame time, that it would have gone thro either of the contiguous ſides, if only one of the forces had acted upon it.

Cor. Hence the force, by which any body moves in a ſtraight line, may be reſolved into two other forces, which will be to each other as the ſides of a parallelogram, of which that ſtraight line is the diagonal.

PROP. V.

The motions and actions of bodies upon each other, in a place that is carried uniformly forward, are the ſame as if that place were at reſt: and any powers or motions that act upon all the bodies ſo as to produce equal velocities in them, in the ſame or in parallel lines, have no effect on their mutual actions or relative motions.

SECT. II.

OF THE CENTRE OF MAGNITUDE, MOTION, AND GRAVITY.

PROP. I.

The centre of magnitude, the centre of motion, and the centre of gravity, may be in the ſame, or in different points of the ſame body.

PROP. II.

The centre of gravity in a homogeneous circular plane body, in a homogeneous globe, cylinder, parallelogram, and priſm, is the ſame with the centre of magnitude.

PROP. III.

The centre of gravity in a homogeneous triangular plane body, is at the diſtance from the vertex, of two thirds of the line biſecting the oppoſite ſide. In a homogeneous trapezium, at the point of interſection, of lines joining the centres of gravity of the triangles, into which the trapezium is divided. And in a homogeneous cone and pyramid, at the diſtance of three fourths of the axis from the vertex.

Of the centres of gravity in bodies of irregular figures, or of heterogeneous parts; and the manner of finding their centres of gravity by experiment.

PROP. IV.

The common centre of gravity of two bodies, is in the ſtraight line joining their reſpective centres of gravity; and the diſtance of the centres of gravity of the two bodies from their common centre of gravity, is reciprocally as their quantities of matter.

PROP. V.

The common centre of gravity of three bodies may be found in this manner. Take the common centre of gravity of two; conſider the two bodies of which that point is the common centre of gravity as one body, then find the common centre of gravity of this compound body, and of the third body; and that point will be the common centre of gravity of the three.

In the ſame manner we may find the centre of gravity of four, of five, or of any greater number of bodies.

PROP. VI.

If a line is drawn from the centre of gravity of a body at right angles to the horizon, and if this line falls within the baſe upon which the body reſts, the body will ſtand; but if without, it will overſet or fall.

Cor. 1. If the centre of gravity of a body, or of a ſyſtem of bodies, be ſupported, the whole body, or the whole ſyſtem of bodies will be ſupported; and hence their centres of gravity may be conſidered as their places, or the parts of ſpace which they occupy.

Cor. 2. The larger the baſe is upon which a body ſtands, and the farther within it the perpendicular drawn from the centre of gravity falls, the firmer the body will ſtand: and the leſs the baſe, or the nearer to its extremity the perpendicular from the centre of gravity falls, ſo much the more eaſily may the body be overſet.

This is equally true of many pieces of heterogeneous matter, when united in ſuch a manner as to make one body.

Cor. 3. Hence the reaſon why bodies of the ſame baſe, but unequal heights, ſtand, or fall, upon the ſame declivity; why ſome tumble, others roll, and others ſlide.

Cor. 4. A double cone, the baſe of which has room to ſink in an inclined plane, will roll up that plane, if the right ſine of the angle of its elevation be leſs than half the diameter of the baſe of the cone.

Cor. 5. A cylinder in which the centre of gravity, and the centre of magnitude are not the ſame, will roll up an inclined plane, if the right ſine of the angle which the plane makes with the horizon, is leſs than the diſtance of the centre of magnitude from the centre of gravity, the ſemidiameter of the cylinder being radius.

Cor. 6. The centre of gravity and the centre of motion, of a mariner's compaſs, of a lamp, of a marine chair, of a time-keeper, may be ſo adjuſted, that they will anſwer the deſign of their conſtruction as machines, tho a conſiderable and irregular motion be given to the bodies on which they are placed.

PROP. VII.

If two or more bodies move equably in any given ſtraight lines, their common centre of gravity will either be at reſt, or move equably in the ſame ſtraight line.

PROP. VIII.

The ſum of the motions of ſeveral bodies in any given direction, is the ſame as the motion of all the bodies in the ſame direction, moved with the velocity of the common centre of gravity.

Of the centre of gravity in quadrupeds, birds, and men, when ſitting, ſtanding, at reſt, in motion, with or without burdens, and in a variety of attitudes that require agility or ſtrength.

PROP. IX.

Tho animals have the power of changing their centres of gravity, according to the conſtruction of their bodies, and according to habits acquired; yet they are always ſubject to the laws above explained.

SECT. III.

OF THE DESCENT OF HEAVY BODIES UPON INCLINED PLANES.

PROP. I.

A heavy body deſcends upon an inclined plane with a motion equably accelerated; allowance being made for the reſiſtance of the air, and the friction of the plane.

COR. I. The velocities of a heavy body deſcending upon an inclined plane are as the times. The ſpace paſſed over is the half of that which might have been uniformly paſſed over in the ſame time with the velocity laſt acquired. The ſpaces are in the duplicate ratio of the times or celerities. And the celerities and times are in the ſubduplicate ratio of the ſpaces. See GRAVITATION, Sect. I. Prop. V, VI, VII.

COR. 2. The aſcent of a heavy body upon an inclined plane is a motion equably retarded.

PROP. II.

The time of a heavy body's deſcent upon an inclined plane, is to the time of its deſcent in a perpendicular, drawn from the height of the plane to a line drawn parallel to the horizon, thro the lower end of the plane; as the length of the plane is to the length of the perpendicular.

COR. The times of deſcent upon different planes of the ſame height are as the lengths of the planes.

PROP. III.

If from the upper end of an inclined plane there be drawn a line at right angles to the horizon, and if from any point of that line there be drawn a perpendicular to the plane; a heavy body will deſcend upon the plane from its higheſt point to that perpendicular, in the ſame time that another heavy body will fall from that point, to the point from which the perpendicular was drawn.

COR. 1. Hence we may determine the ſpace thro which any heavy body will fall in a perpendicular, while another falls thro a given ſpace in an inclined plane. And if there are two or more inclined planes, and a ſpace is taken in the firſt which is paſſed over by a heavy body in a given number of vibrations, is to the ſquare of 60, or the number of ſeconds in a minute; ſo is the length of the ſtandard pendulum, or 39.128 inches, to the length of the pendulum ſought.

PROP. IV.

Tho the eye cannot perceive any difference, in the times of the oſcillations of the ſame pendulum thro ſmall arches, yet there are differences; and they, in a great number of oſcillations, make conſiderable errors in meaſuring time.

OF THE CYCLOID.

PROP. V.

The oſcillations of a pendulum moving in a cycloid are performed exactly in equal times, in large arches as well as in ſmall: but if the vibrations of a pendulum, without a cycloid, are in ſmall arches, they will all be performed in the ſame time nearly; the cycloidical curve and the circle being nearly the ſame for a little way at the loweſt point.

OF THE ROD PENDULUM.

PROP. VI.

If a rod, or cylindrical piece of matter, be ſuſpended at one end, and put into motion; it will oſcillate in equal times with a common pendulum, the length of which is two thirds of the length of the rod.

Cor. 1. A rod or bar oſcillates in the ſame manner as it would do, if the gravity of all its parts were contracted into a point at two thirds of its length from the pin; which point is called the centre of oſcillation.

COR. 2. If any obſtacle ſtops the centre of oſcillation, the whole motion of the bar will be deſtroyed at once; and, conſequently, the whole of its force will be received by that obſtacle.

COR. 3. If any other part of the bar ſtrikes againſt an obſtacle, the centre of oſcillation will tend forwards, and by that tendency break or bend the rod; if the force impreſſed be ſufficiently great.

COR. 4. Hence it is eaſy to explain the centre of percuſſion; or that part of an inſtrument, or weapon, which will ſtrike with the greateſt force.

COR. 5. All oſcillating bodies, of what ſhape ſoever, have their proper centres of oſcillation. If the ſtring, or wire, of a common pendulum be ſmall, and the ball large; its centre of oſcillation will be within the ball, and near its centre of gravity.

COR. 6. The length of a pendulum muſt always be eſtimated by the diſtance between the centre of ſuſpenſion, and the centre of oſcillation.

PROP. VII.

If a pendulum, which ſwings ſeconds at the equator, is 39 inches in length; the following table ſhews the ſeconds it will gain in twenty-four hours at different latitudes; and the lengths to be added to it, in order to make it ſwing ſeconds at thoſe latitudes.

Lat.	Seconds gained.	Inches to be added.
10	6.9	0.0062
20	26.7	0.0246
30	57.1	0.0516
40	94.3	0.0853
50	134.0	0.1212
55	153.2	0.1386
60	171.2	0.1549
70	201.6	0.1824
80	221.4	0.2033
90	228.3	2.2065

Cor. Theſe differences ariſe from the figure of the earth, and from the different degrees of centrifugal force communicated by the earth's motion to a pendulum in different latitudes. See GRAVITATION, Sect. I. MECHANICS, Sect. VII.

SECT. V.

OF MEASURES AND WEIGHTS.

Of the advantages which would ariſe to commerce, and to philoſophy, from general ſtandard meaſures, and weights.

Without changing common names, meaſures of length and ſurface might be taken from a pendulum, as a ſtandard, that vibrates ſeconds under the equator; or in a certain latitude: and this univerſal ſtandard might be divided and ſubdivided into decimal parts, with great accuracy and minuteneſs. From the univerſal ſtandard, local ones might be made of bars of copper, having the ſame denſity and varniſh; and the ſame degree of heat when uſed. Meaſures of capacity might be cubes of the ſame univerſal ſtandard, and of its diviſions and ſubdiviſions. And weights might be cubes of copper of the ſame ſtandard; or of rain-water, when of a certain temperature.

Of ſome of the meaſures, and weights, that are moſt commonly uſed.

British Meaſures of Length, and Superficies.

Barley Corns.						
3	1 Inch.					
36	12	1 Foot.				
108	36	3	1 Yard.			
594	198	16½	5½	1 Pole.		
23760	7920	660	220	40	1 Furl.	
190080	63360	5280	1760	320	8	1 Mile

Forty poles in length, and four in breadth, make a ſtatute acre; or, 4840 ſquare yards.

Before England and Scotland became one kingdom, the Scotch inch and foot were longer than the Engliſh, in the proportion of 196 to 195. The Scotch mile was 5920 feet. And the Scotch and Engliſh acre, ſuppoſing the feet in both meaſures alike, were nearly as 5 to 4; but if the difference of the feet is taken into the account, as 10,000 to 7,869.

British Meaſures of Fluids; or, Liquid Meaſures.

Cubic Inches.						
231	1 Gallon.					
9702	42	1 Terce.				
14553	63	1½	1 Hogſhead.			
19404	84	2	1⅓	1 Puncheon.		
29106	126	3	2	1½	1 Butt or Pipe	
58212	252	6	4	3	2	1 Tun.

The Scotch pint contains 105 cubic inches, and the quart 210; which, therefore, is about one tenth leſs than the Engliſh wine gallon, which contains 231.

Scotch Gills.					
4	1 Muchkin.				
8	2	1 Chopin.			
16	4	2	1 Pint.		
32	8	4	2	1 Quart.	
128	32	16	8	4	1 Gallon.

British Meaſures of Solids; or, Dry Meaſures.

Cubic Inches.				
268.8	1 Gallon.			
537.6	2	1 Peck.		
2150.4	8	4	1 Buſhel.	
17203.2	64	32	8	1 Quarter.

The ſtandard for meaſuring corn, &c. is the Wincheſter gallon, which contains 268.8 cubic inches; and the Wincheſter buſhel, which contains eight gallons, or 2150.4 cubic inches. Eight buſhels make a quarter; four quarters, a chaldron; five quarters, a load; and ten quarters, a tun.

There was formerly in Scotland but one meaſure of capacity for ſolids; and ſome commodities were ſold by the exact meaſure of the ſtandard, and others were heaped. But the laſt method was diſcharged by law, becauſe liable to abuſe; and larger meaſures were appointed in its place. Hence meaſures, of the ſame denomination, for different things, are very different. Thus, the barley firlot, or four pecks, contains 3208 cubic inches; and the wheat firlot, or four pecks of wheat, contains 2199 cubic inches; or about a hundredth part more than the Wincheſter buſhel.

BRITISH WEIGHTS.

TROY WEIGHT.

Grains.			
24	1 Penny-weight.		
480	20	1 Ounce.	
7560	240	12	1 Pound.

The Troy weight was formerly uſed for every purpoſe; but it is now chiefly uſed for comparing different weights with each other; for making experiments in Natural Philoſophy; and for weighing gold, ſilver, jewels, &c.

AVERDUPOIS WEIGHT.

Drachms.				
16	1 Ounce.			
256	16	1 Pound.		
28672	1792	112	1 Hundred.	
573440	35840	2240	20	1 Tun.

The pound Averdupois contains 7000 grains Troy. The ounce was formerly divided into eight drachms, and the drachm into three ſcruples; but the preſent diviſion is into 16 drops or drachms. A ſtone is equal to 14 pounds, and 28 pounds to a quarter of a hundred.

The Trone weight is the moſt antient of all the kinds uſed in Scotland; but it has long been prohibited by law. The Scotch Troy weight was the ſame with that of Paris and Amſterdam; and is known by the name of Dutch weight. The Scotch Troy pound contains 7600 Britiſh Troy grains; and is equal to 17 ounces, 6 drachms, Averdupois. The pound contains 16 ounces; and 16 pounds make a ſtone.

The variations, in different counties, are very great, both in weights and meaſures.

A Table of Meaſures, reduced to the Paris fathom or toiſe, foot, inch, line, and decimal parts of a line; according to the ſtandard of the Royal Academy of Sciences; which is the moſt accurate, and the beſt known in Europe.

	Toiſe	Inch	Line	Dec.
The Engliſh foot	0	11	3	1154
The Rhindland foot	0	11	7	183
The Bologna foot	0	14	0	60
The Swediſh foot	0	10	11	75
The Chineſe royal foot	0	11	9	9
The antient Roman foot	0	10	10	90
The antient Greek foot	0	11	4	56
The Arabian foot	0	9	10	72
The Alexandrian foot	0	13	2	9
The Engliſh mile	830	0	0	0
The modern Roman mile	764	0	0	0
The antient Roman mile	766	0	0	0
The Italian geograph. mile	958	0	0	0

A Table of ſome foreign weights, uſed in making experiments in Natural Philoſophy, compared with the Engliſh pound Averdupois.

	Averd. ℔.
100 pounds of Paris is equal to	$108\frac{1}{2}$
100 ℔ of Lyons	$93\frac{1}{4}$
100 ℔ of Rome	$78\frac{1}{2}$
100 ℔ of Turin	$71\frac{3}{4}$

	Averd. ℔.
100 ℔ of Amſterdam is equal to	$108\frac{1}{2}$
100 ℔ of Geneva	$121\frac{1}{2}$
100 ℔ of Sweden	$92\frac{3}{4}$
100 ℔ of Ruſſia	$88\frac{1}{4}$

SECT. VI.

OF PROJECTILES.

PROP. I.

If a body is projected in an oblique or horizontal direction, its path will be a parabola, when there is no reſiſtance from the air.

Of the variation from the parabolic curve, in the motion of projectiles, occaſioned by the reſiſtance of the air.

PROP. II.

When the impetus of a projected body, or the charge, is the ſame; the greateſt amplitude, or range, is made with an angle of elevation of forty-five degrees.

PROP. III.

The amplitudes are equal, which are made with any angles of elevation, at equal diſtances from half a right angle, if the impetus be the ſame.

Cor. Hence, of two elevations, that one may be taken, which is the moſt convenient for hitting a mark.

PROP. IV.

With the ſame angle of elevation, the amplitude of a projected body is as the ſquare of the impetus, or charge.

PROP. V.

The greateſt altitude, or height, of a perpendicular projection, is equal to half the greateſt range.

PROP. VI.

If the impetus of a projectile be taken as the diameter of a circle, and a perpendicular be drawn from the circumference to the diameter, from the point where the line of elevation meets the circumference; the diſtance of that point from the horizontal plane will be the greateſt height to which the projectile will aſcend; and four times that perpendicular will be the utmoſt range.

Cor. If the amplitude and direction are given, the impetus and altitude may be found: if the impetus and amplitude are given, the direction and altitude may be found: and, in general, every thing relating to the impetus, elevation, amplitude, or path, of the projectile, may be found; whether the object to be ſtruck is in the ſame horizontal plane with the point from which the body is thrown; or above that plane, or below it.

SECT. VII.

OF CENTRAL FORCES.

PROP. I.

Centrifugal and centripetal forces, acting upon the ſame body, may overcome each other; or they may be in ſuch proportions, that the body acted upon will revolve in the ſame path.

PROP. II.

When a body, that is acted upon by a centrifugal and centripetal force, recedes from the centre by the firſt force, or accedes to it by the ſecond, its path will be a ſpiral line: if the firſt is ſuſpended, the body will move towards the centre; if the ſecond is ſuſpended, it will fly off in a tangent.

PROP. III.

A body will revolve in an ellipſe, by the compoſition of a projectile force and centripetal force acting upon it; and if a ſtraight line is drawn from the body, to the focus of the ellipſe, that line will deſcribe areas proportional to the times in which the body moves.

PROP. IV.

If two bodies are moved by centrifugal and centripetal forces; and if their periodic times, their quantities of matter, and diſtances from the centre are equal; their centrifugal forces will be equal.

PROP. V.

If their periodic times, and quantities of matter are equal, but their diſtances unequal; their centrifugal forces will be as their diſtances.

PROP. VI.

If the diſtances, and periodic times, are equal, but the quantities of matter unequal; the centrifugal forces will be as the quantities of matter.

PROP. VII.

The centrifugal forces of ſuch revolving bodies are in direct proportion to their quantities of matter, multiplied into their reſpective velocities, or into their diſtances from the centres of their reſpective orbits.

Cor. If the periodic times are equal, and the diſtances from the centre reciprocally as the quantities of matter, the centrifugal forces will be equal.

PROP. VIII.

If the quantities of matter, and diſtances from the centre, are equal, but the periodic times unequal; the centrifugal forces will be reciprocally as the ſquares of the periodic times; or as the ſquares of the velocities.

PROP. IX.

If the quantities of matter are equal, and the periodic times unequal; but ſuch, that the ſquares of the times are as the cubes of the diſtances, the centrifugal forces will be inverſely as the ſquares of the diſtances.

PROP. X.

If two or more bodies are joined together, or connected by any force; and if they revolve round each other, they will revolve round their common centre of gravity, as their common centre of motion.

PROP. XI.

If any number of bodies are carried round in a vortex, the matter of which is denſer than that of the bodies, they will run into the centre in a ſpiral line: if it is not ſo denſe, they will recede from the centre in a ſpiral line: and if it is of the ſame denſity, they will have their periodic times equal, whatever their diſtances are.

PROP. XII.

If a motion be given to a terraqueous ſphere, ſuch as that its centrifugal force can overcome its centripetal force, it will become oblate at the poles.

SECT. VIII.

OF THE MOMENTUM AND COLLISION OF BODIES.

PROP. I.

The momentum of a body is in a ratio compounded of its quantity of matter and velocity,

COR. 1. The momentums of different bodies are to each other, in the ſame proportion as the products got by multiplying their reſpective quantities of matter into their velocities.

COR. 2. A very ſmall body may have a momentum equal to that of a very great body; if their quantities of matter, and celerities are reciprocally proportional.

PROP. II.

The momentums of bodies may be the ſame, and yet their effects may be very different; according to their ſhapes and bulks; according to the ſhapes, bulks, and coheſions, of the bodies on which they act; and according to the times in which their forces are ſpent.

OF HARD, NON-ELASTIC, AND ELASTIC BODIES.

PROP. III.

There are no bodies on which experiments have been made, that are perfectly hard, perfectly non-elaſtic, or perfectly elaſtic.

PROP. IV.

If a non-elaſtic body impinges directly on a non-elaſtic body which is at reſt, or moving with leſs velocity the ſame way, the ſum of the momentums will be the ſame both before and after the impulſe.

PROP. V.

When two non-elaſtic bodies impinge directly on each other with contrary motions, the ſum of the momentums, after the impulſe, will be equal to the difference of the momentums, before the impulſe.

PROP. VI.

If the ſum of two conſpiring motions, or the difference of two contrary motions, be divided by the ſum of the quantities of matter in non-elaſtic bodies, of which the colliſion is direct; the quotient will give their common velocity after the ſtroke. And if the velocity after the ſtroke be multiplied into the quantity of matter in each body, the products will expreſs the momentum of each body after the ſtroke.

PROP. VII.

If a non-elaſtic body ſtrikes directly againſt an immoveable obſtacle, the whole motion of the body will be deſtroyed by the ſtroke; and, conſequently, the whole of its momentum.

PROP. VIII.

When elaſtic bodies are ſtruck, they change their figure, and recover it again; and the elaſtic power of the ſame body exerts itſelf in one conſtant proportion to the compreſſing force.

Cor. The reſiſtance of elaſtic bodies is proportional to the ſpace thro which they are forced.

PROP. IX.

When two equal and elaſtic bodies impinge directly againſt each other with equal velocities, they will recede from each other, after the colliſion, with the ſame velocities: and if one of them in motion impinges againſt the other at reſt, it will communicate its whole velocity to the other, and remain at reſt itſelf.

PROP. X.

If two equal and elaſtic bodies, are moved in the ſame direction, but with unequal velocities; after the ſloweſt moving body has received an impulſe from the other, they will exchange velocities, and continue in the ſame direction as before: and if they meet each other with different velocities, they will exchange velocities, and move, after the impulſe, in contrary directions.

PROP. XI.

In order to determine the velocity of unequal elaſtic bodies after the impulſe; find the common velocity with which the bodies would move, after the ſtroke, if they had been non-elaſtic; and find alſo the velocity loſt or gained: then ſubtract the velocity loſt from the common velocity; or, to the common velocity, add the velocity gained; and the difference, or the ſum, will be the velocity ſought: but, if the velocity loſt be greater than the common velocity, ſubtract the common velocity from the velocity loſt, and the remainder will give the velocity ſought, in a contrary direction.

PROP. XII.

If an elaſtic body impinges againſt an immoveable obſtacle, it will rebound with the ſame velocity that it came.

PROP. XIII.

If there is a ſeries of elaſtic bodies; and if one, or two, or more, at one extremity of the ſeries, are impelled againſt the reſt; an equal number will fly off, and with the ſame velocity, from the other extremity.

PROP. XIV.

In the oblique percuſſions of elaſtic bodies, the angle of incidence is equal to the angle of reflection. The force of the oblique percuſſion, is to that of the direct, as the ſine of the angle of incidence is to the radius: it may be reſolved into two impulſes; the one direct, and the other parallel to the tangent, at the point of impulſe: the effect of the firſt may be computed by the above rules; and the effect of the laſt will continue, after the colliſion, the ſame that it was before.

SECT. IX.

OF THE SIMPLE MECHANIC INSTRUMENTS.

OF THE LEVER.

PROP. I.

When a power or force is applied perpendicularly to a lever, it will ſuſtain a weight, or impediment, acting perpendicularly againſt the lever; if the power and weight are reciprocally to each other as their diſtances from the prop or ſupport: and if the power is increaſed, it will give motion to the weight; or, if the weight is increaſed, it will give motion to the power: and that, whether the prop is placed between the weight and the power; or, the weight is placed between the power and the prop; or, the power is placed between the prop and the weight.

Cor. 1. Hence there are three kinds of levers. The firſt, when the prop is between the power and the weight; the ſecond, when the weight is between the power and the prop; and the third, when the power is between the prop and the weight.

Cor. 2. The velocities of the power and weight are as their diſtances from the prop.

Cor. 3. A very ſmall power may raiſe a very great weight; if the length of the lever, and the ſpace to be paſſed thro by the power, can be increaſed in the proportion required by the propoſition.

PROP. II.

If a lever is bent; or is made up of parts, which do not lie in one ſtraight line; the diſtances of the power and weight from the prop, muſt be eſtimated by perpendiculars from the prop to the lines of direction of the power and weight reſpectively.

OF THE STEELYARD.

PROP. III.

In a ſteelyard, the matter on one ſide of the prop, ought to ſuſtain the matter on the other: and the centre of gravity ought to be a little below the centre of motion; and then it is fit for examining weights according to the rule in the firſt propoſition.

OF THE COMMON BALANCE.

PROP. IV.

In a balance, the friction ought to be ſmall; the arms of equal lengths; the ſcales and ſtrings of equal weights; the centre of gravity, and the centre of motion, ought to be different; and the centre of motion ought not to be placed above the points of ſuſpenſion of the ſcales.

Cor. By not obſerving theſe rules, a balance may be made that is inaccurate, or falſe.

Of many common inſtruments, that are levers of the different kinds; and of the bones of animals, which are levers of the third kind.

OF THE WHEEL AND AXLE.

PROP. V.

In the wheel and axle, the power will ſuſtain the weight, when the power is to the weight as the diameter of the axle is to the diameter of the wheel; and if the power is increaſed, the weight will be moved; or, if the weight is increaſed, the power will be moved: their velocities being, reciprocally, as the circumference of the axle to the circumference of the wheel.

Cor. 1. The thickneſs of the rope, if conſiderable, muſt be added to the diameter of the axle: and if a rope is coiled round the wheel, the thickneſs of that rope muſt, in like manner, be added to the diameter of the wheel. Thus, if the rope is coiled once upon the axle, its ſemidiameter muſt be added to the radius of the axle: if twice, three times the ſemidiameter muſt be added: if a third time, five times the ſemidiameter; and ſo on.

Cor. 2. In order to work the wheel and axle, continually, with the ſame advantage; if a rope is coiled upon the axle, and no rope is coiled upon the wheel, the power muſt be increaſed as the rope recedes from the axle; and in the above proportion.

OF THE PULLEY.

PROP. VI.

No advantage is given to the power by a ſingle upper wheel, when conſidered as a mechanic inſtrument: but other advantages are gained; ſuch as, a ſmooth motion to the weight, leſs rubbing to the rope; and, by means of it, a man can do more work, on account of the make of his body.

PROP. VII.

In a combination of upper and lower wheels, where one rope runs thro all the wheels, the power will ſuſtain the weight, when the power, is to the weight, as unity to the number of folds in the rope; the fold to which the power is applied not being taken into the account: or, as unity to twice the number of the lower wheels.

PROP. VIII.

In a combination of wheels, where each lower wheel has its own rope, the power and weight will ſuſtain each other, when the power, is to the weight, as unity to the number two multiplied into itſelf, as often as there are lower wheels; and, conſequently, if either the power or the weight is increaſed, it will prevail.

PROP. IX.

In other combinations of wheels with different ropes, the number two muſt be multiplied into itſelf, as often as there are lower wheels; and the ſinking of the wheels muſt be added to the ſum

Cor. In all combinations of wheels in the pulley, the power and weight will be to each other as their reſpective velocities reciprocally.

Of the advantages, and diſadvantages, of different combinations of wheels; and of the beſt method of fitting up the pulley for uſe.

OF THE INCLINED PLANE.

PROP. X.

If a power, which acts in a direction parallel to an inclined plane, is, to the weight to be raiſed, as the perpendicular height of the plane is to its length, the power will ſuſtain the weight.

Cor. By the inclined plane, weights may not only be raiſed; but by it, as with a lever, a very ſmall power will ſuſtain a very great weight; if they are to each other, as the height of the plane is to its length.

OF THE WEDGE.

All bodies, to which the wedge is applied, may, at a medium, be reduced to three kinds: ſuch as do not fly before the wedge; ſo that every part of the cleft is in contact with it: ſuch as fly ſo much, that no part of the cleft is in contact with it, but at its upper extremity: and ſuch as fly to its middle; ſo that one half of the ſide of the wedge is in contact with part of the cleft. An example of the firſt kind is clay, or ſoft lead; of the ſecond, dry fibrous elaſtic wood; and of the third, ſtones, or ſoft timber.

PROP. XI.

In the ſingle wedge; the power, that is, the momentum of the hammer and wedge, will be equal to the reſiſtance; that is, to the coheſion of the body to be cloven, with the weight of the matter that is moved; and if increaſed, will overcome it; when the power is to the reſiſtance, as the ſquare of the length of the cleft is to the area of the triangle formed by the cleft, at every ſtroke. And in the double wedge; as twice the ſquare of the length of the cleft, to the two triangles formed by the cleft.

Or, if the time in which the effect is produced is not conſidered, and the action of the reſiſtance is at right angles to the ſides; then the whole reſiſtance, on both ſides of the wedge, is to the whole power of the wedge, when they ſuſtain each other, as the ſquare of the ſide of the wedge is to the rectangle, under half the back, and the perpendicular height.

OF THE SCREW.

PROP. XII.

In the ſcrew; if the power is to the weight or reſiſtance, as the diſtance between two of the neareſt threads, meaſured upon the axis, is to the circumference of the circle deſcribed by the power, they will ſuſtain each other; and if the power is increaſed, it will move the weight; and on the contrary.

Cor. If the diſtance between two of the neareſt threads is diminiſhed, the power will be increaſed; or, if the diſtance is increaſed, the power will be diminiſhed; and in exact proportion.

OF THE PERPETUAL SCREW.

PROP. XIII.

In the perpetual ſcrew and wheel; if a weight acts at the circumference of the wheel, the power will ſuſtain the weight, when the power is to the weight, as the diſtance of the teeth or threads, to the path deſcribed by the power.

PROP. XIV.

All the mechanic inſtruments are not equally fit for producing the ſame effect; ſo that a choice of them muſt be made, according to the effect required.

Tho the power may be increaſed by the application of a mechanic inſtrument, yet a greater effect cannot be produced by it, in the ſame time; but the time muſt be increaſed in the direct ratio of the increaſe of the power.

In eſtimating the power; the weight of the matter of which the inſtrument is made, and the friction of its parts, muſt be added or ſubtracted, according as it is for, or againſt, the power.

SECT. X.

OF FRICTION.

PROP. I.

The ſofter, or rougher, the bodies are which rub upon each other, the greater, in general, is their friction. If their preſſure is increaſed, their friction will be increaſed in the ſame ratio, all other circumſtances being equal. If their velocity is increaſed, their friction will be increaſed; but not in the ſame proportion: and the increaſe and diminution of the ſurfaces makes, in general, no change in the friction, while the preſſure and velocity are the ſame; but there are variations from this, in particular caſes.

PROP. II.

Wood ſlides more eaſily upon the ground, or earth, in wet weather than in dry; and more eaſily than iron, in dry weather; but iron more eaſily than wood, in wet weather. A cubic piece of ſmooth ſoft wood, eight pounds in weight, moving upon a ſmooth plane of ſoft wood, at the rate of three feet every ſecond, has a friction equal to about two thirds of its weight: ſoft wood, upon hard wood, has a friction equal to one ſixth part of its weight: and hard wood, upon hard wood, has a friction equal to about one eighth of its weight.

PROP. III.

In wood rubbing upon wood; oil, or greaſe, properly applied, makes the friction about two thirds leſs. Wheel naves, when greaſed, have four times leſs friction than when wet.

PROP. IV.

When poliſhed ſteel moves on ſteel, or pewter, properly oiled, the friction is about one fourth of the weight; on copper, or lead, one fifth of the weight; on braſs, one ſixth; and metals have more friction, when they move on metals of the ſame kind, than when they move upon different metals.

PROP. V.

The friction of ropes moving upon machines, differs according to their ſtiffneſs, their greaſe, and the ſtate of the weather. All circumſtances being equal, the difficulty of bending a rope, is as the ſquare of its diameter, and its tenſion directly; and reciprocally as the diameter of the cylinder or pulley about which it goes. A rope of one inch in diameter, the tenſion of which is made by five ℔ over a pulley three inches in diameter, requires the weight of one ℔ to bend it.

PROP. VI.

In the ſimple lever, the friction is eaſily eſtimated by the above experiments.

In the wheel and axle; the friction on the axle is as the weight, the diameter of the axle, and the velocity.

In the pulley; the friction on the axis is as the weight, the velocity, the diameter of the axis directly, and the diameter of the pulley inverſely. A power of 100 ℔, with the addition of 50 ℔, will raiſe 500 ℔ with a tackle of five ropes; and 15 ℔ over a ſingle pulley will raiſe only 14 ℔. The friction is great when the ſheves rub againſt the blocks, or when the axis and hole are much worn.

PROP. VII.

Friction on the inclined plane, is according to the matter, and ſurface, of which the plane and weight are made, and the rubbing of the rope and wheel as above explained.

In the wedge, and common ſcrew; the friction is great; and is, at leaſt, equal to the power, when they retain the poſition which they have gained.

PROP. VIII.

In order to eſtimate the friction of any machine; begin at the power, conſider the velocity, and the weight at the firſt rubbing part, and eſtimate its quantity of friction as above. Proceed in the ſame manner with the next rubbing part, and ſo thro the whole, making an allowance for the increaſe of friction by every addition to the power.

Of friction wheels; and other means of diminiſhing friction, beſides oil and greaſe.

SECT. XI.

OF COMBINED MECHANIC INSTRUMENTS; OR MACHINES.

PROP. I.

In a combination of levers; the power will ſuſtain the weight, when the power is to the weight, as the diſtance of the weight from the next ſupport, is to the powers of all the levers multiplied into each other.

PROP. II.

In a combination of wheels; the power will ſuſtain the weight; if the power be to the weight, as the product of the diameters of all the axles, leaves, pinions, or trundles, to the product of the diameters of all the wheels.

PROP. III.

In every machine, the power will ſuſtain the weight, when the velocity of the power is to the velocity of the weight, as the weight to the power.

COR. If a weight is moved by a machine, what is gained in power is loſt in time: and if a power is given, a greater weight may be moved with a leſs velocity; or, a leſs weight with a greater velocity; but to do otherwiſe is impoſſible.

PROP. IV.

If the power which moves a machine acts entirely within it, and exerts its force againſt an external object; then the power to move the machine will be the ſame as if the machine were at reſt, and the object were to be moved; ſuppoſing the object to be as eaſily moved as the machine.

COR. 1. If the external power to move any machine be given; and if the machine is to be moved by an internal power; that power may be found by ſuppoſing the machine to be at reſt, and the object to require the ſame power to give it motion.

COR. 2. If the power acting within a machine be not communicated to ſome external object, it will have no power to move the machine.

SECT. XII.

OF MODELS.

Of models for giving a diſtinct conception of machines.

Of working models, or machines in miniature.

A working model may be conſtructed ſo as to ſatisfy the expectations of the inventor; and yet when a large machine is made with the ſame proportions, it may not work, or it may break in pieces; becauſe the friction, or the relative ſtrength of the parts, may not be the ſame that they were in the model.

SECT. XIII.

OF THE STRENGTH OF TIMBER, AND ROPES.

PROP. I.

The lateral ſtrength of any piece of timber; the ſection of which is a rectangle, is directly as the breadth and the ſquare of the depth.

PROP. II.

In ſquare timber, the lateral ſtrength is as the cube of the breadth, or depth.

PROP. III.

If a weight be laid upon a ſtraight beam, ſupported at both ends; its bending will be nearly as the weight and length directly; and as the breadth, and ſquare of the depth reciprocally.

PROP. IV.

If any force be applied laterally to a beam, one end of which is fixed in a wall, the ſtreſs upon any place will be directly as the force, and its diſtance from the wall.

PROP. V.

If two beams are placed, ſo as to make the two ſides of an iſoceles triangle, of which the baſe is in a horizontal plane; if a weight is equally diffuſed over them, or laid upon their middle; the ſtreſs upon the beams will be as their lengths, as the coſines of the angles which they make with the horizon, and as the weights.

Cor. From theſe propoſitions we may ſee how to ſave timber, and gain ſtrength, in erecting machines, and building houſes and ſhips.

PROP. VI.

A piece of good oak, taken lengthwiſe, without knots, an inch ſquare and a yard long, and ſupported at both ends, will bear in the middle, 330 ℔; but it ſhould not be loaded with above a third part of that weight, for any length of time.

Wood is weaker when green than when dry: the wood of the branches is weaker than that of the trunk; and the heart of the tree, when freſh and ſound, is the ſtrongeſt of all.

The relative ſtrength of the different kinds of wood is, at a medium, as follows: oak as 12; yew as 11; aſh as 10; elm, as 9; walnut, as 8; fir, alder, plane, as 7; beech, hazel, birch, willow, as 6, &c.

Cor. Hence machines cannot be increaſed to any bigneſs: joiſts of a certain length would break: trees, and animals, of a certain magnitude, would fall to pieces, by their own weight.

PROP. VII.

A rope of hemp, an inch in circumference, will bear 100 ℔; and the increaſe of the ſtrength of ropes, if very good, is nearly as the ſquares of their diameters; but neither beams, nor ropes, ſhould bear, for a long time, more than a third part of what they will eaſily bear, when freſh, and newly made.

Of fixing ropes in machines; and of the beſt method of making knots.

SECT. XIV.

OF POWERS APPLIED TO MACHINES.

PROP. I.

If the moving force is a weight, a ſpring, water, or air, the laws by which they act are aſcertained in Gravitation, Mechanics, Hydroſtatics, and Pneumatics.

PROP. II.

A man, of ordinary ſtrength, turning a roller by the handle, can work for a whole day againſt a reſiſtance equal to 30 ℔. If he works ten hours in a day, he will raiſe that weight three feet and an half, in one ſecond of time: or if the weight is greater, he will raiſe it ſo much leſs in proportion. He may act for a ſhort time againſt a reſiſtance of

50, 60, or 70 ℔. If the weight of his own body is 140 ℔, he can draw about the half of that weight horizontally. If he is accuſtomed to bear loads, he will carry from 200 to 250 ℔; according to the manner in which he is loaded.

If two men work at a windlaſs, or roller, they will more eaſily raiſe 70 ℔, than one man will raiſe 30 ℔; provided the elbow of one of the handles is at right angles to that of the other; and with a fly or wheel applied, they will do a third part more work than without one; unleſs the machine is better made than thoſe that are in common uſe.

PROP. III.

The ſtrength of one good horſe is commonly equal to that of five men; and he will carry from 240 to 270 ℔. He will draw to greateſt advantage, when the line of draught makes a ſmall angle with the horizon, and when he draws by his breaſt. In a gin he can draw 200 ℔ for eight hours in a day, at the rate of two miles and a half every hour. If he draws 240 ℔, he can work but ſix hours, and not go ſo faſt. In a cart, he can draw 1000 ℔. The worſt way of applying his ſtrength is to make him carry, or draw, up a ſteep hill; becauſe three men, each carrying 100 ℔ up a hill, will go faſter than a horſe with 300 ℔. Tho a horſe will draw in a circular path eighteen feet in diameter, yet he ought not to work long in one that is leſs than thirty feet in diameter, on account of the diſadvantageous poſition given to his body in a ſmall circle.

Of the ſtrength of oxen and mules; and of the beſt way of applying their ſtrength.

SECT. XV.

OF MACHINES FOR WAR.

Of the ſword, ſpear, dart, javelin, bow, arrow, ſling, &c.

Of bucklers, helmets, coats of mail, &c.

The battering ram, baliſta, and catapulta, of different kinds, explained by working models.

PROP. I.

The momentum of a battering ram may be equal to that of a cannon ball; and the momentum of ſtones, &c. thrown by the engines of the ancients, may be equal to the momentum of bombs.

Of the teſtudo, muſculus, crates, pluteus, vinea, &c.

Of the fortified towns of the ancients. Of their attack and defence.

The muſket, cannon, &c.; mortars, bombs, carcaſſes, &c. explained by models.

Of faſcines, mantlets, &c. and the implements of attack and defence.

The fortified towns of the moderns; and their attack and defence explained by models.

SECT. XVI.

OF MACHINES FOR AGRICULTURE.

The ſpade, plough, harrow, &c. of different kinds, and the methods of uſing them to moſt advantage explained by models.

SECT. XVII.

OF WHEEL-CARRIAGES.

Of the conſtruction of carts, waggons, and coaches.

PROP. I.

A carriage is drawn on a horizontal plane with the leaſt force, when the wheels are of ſuch a ſize, that the line of draught is parallel to the plane.

Cor. Hence, if a line is drawn parallel to the plane, thro the higheſt point in the line of draught; and if a perpendicular is drawn from that line to the plane, and if the ſemi-diameter of the wheels is leſs than the perpendicular, the larger the wheels, the more eaſily will the carriage be drawn; and, on the other hand, the leſs the wheels, the more eaſily will the carriage be drawn, if the ſemidiameter of the wheels is greater than the perpendicular.

PROP. II.

A carriage is more eaſily drawn with four equal wheels, than when the two fore wheels are ſmaller than the two hind wheels.

PROP. III.

If the fore wheels are leſs than the hind wheels, a carriage will be drawn with much greater eaſe, when the load is behind, than when before.

PROP. IV.

If a carriage is drawn up a hill, or upon an inclined plane, the power will ſuſtain the carriage, when the power, or force, is to the weight, or carriage; as the height of the plane, or the ſine of the angle which the plane makes with the horizon, is to the length of the plane.

Of the advantages and diſadvantages of the carriages that are in common uſe, and of broad wheels, upon different kinds of roads.

Of the beſt method of making roads; and of the great diſadvantage of making ſtraight-line roads for carriages, in a hilly country.

SECT. XVIII.

OF MACHINES FOR ARCHITECTURE.

Of the architecture of different ages, and countries.

Of the foundations of buildings, according to the nature of the ground. Of piles for foundations; and working models of engines for driving them. Of walls. Of roofs. Of arches; and the principles of their conſtruction.

Of piers. Of bridges; and the beſt method of conſtructing them.

PROP. I.

The piers of a bridge may be built upon platforms, and piles, laid beneath the bed of a river, with every conveniency for the men to work, and full time for the cement to dry; tho the courſe of the river ſhould not be changed.

SECT. XIX.

OF MACHINES FOR MANUFACTURES.

Of the ſpindle and diſtaff. Of the ſpinning wheel. Of the twiſting mill. Of looms; and a variety of machines for manufactures.

SECT. XX.

OF MACHINES FOR MEASURING TIME.

Of the clepſydra, and lamp, &c. of the ancients.

Of the ſand-glaſs, candle, &c. of the moderns.

Clocks, watches, time-keepers, and their various improvements, explained by models.

SECT. XXI.

OF SPHERES OF THE WORLD.

Of the various contrivances of armillary ſpheres, globes, &c. for explaining the phenomena of the heavens.

Of the ſphere, planetarium, or orrery, which was made by Archimedes; and of ſimilar machines by Poſidonius, and Severinus Boethius, &c.

Of planetariums moved by clock-work. Of thoſe at Lunden, Straſburgh, Lyons, &c.

Of the orreries made in London in the beginning of the eighteenth century, and at preſent uſed in the different countries of Europe.

SECT. XXII.

OF PERPETUAL MOTIONS.

Of various machines that have been contrived in order to move continually; without an external power acting upon them. Of the cauſes of the want of ſucceſs in ſuch attempts; and of the uſeleſsneſs of ſuch machines, tho they could be made in the way commonly propoſed.

HYDROSTATICS.

SECT. I.

OF THE COMPRESSIBILITY OF WATER.

PROP. I.

Common water may be conſidered, without any error in Hydroſtatics, as an incompreſſible fluid: for its diminution in bulk by preſſure is very inconſiderable; it being leſs than one part of 10,000, in an ounce of water, when preſſed by a force equal to 14 lb.

COR. The laws to which water is ſubject, may be applied to other incompreſſible fluids; allowance being made for the difference of their denſities.

SECT. II.

OF THE WEIGHT, AND PRESSURE OF WATER.

PROP. I.

Water weighs, or gravitates, in water; or in proprio loco; as well as in air, or in vacuo.

PROP. II.

The ſurface of water, when at reſt, is parallel to the horizon.

This is only true with regard to our ſenſes, and with regard to ſuch quantities of water as are in a jar, pond, &c.; for the ſurface of water in ſmall veſſels is ſenſibly convex, or concave; according as the ſides of the veſſels are dry or wet; and the ſurface of a drop of water, and of the ocean, is ſenſibly ſpheroidical.

Cor. Hence the ſurface of water is employed for making planes parallel to the horizon: and hence the uſe of fluid levels; that is, of water or alcohol encloſed in glaſs tubes, with a bubble of air, and an apparatus for giving ſmall elevations.

PROP. III.

The parts of water preſs each other; and they preſs, at the ſame depth, with the ſame force, in every direction.

PROP. IV.

If water is at reſt in any veſſel, the baſe of which is parallel to the horizon; the preſſure of the water upon the baſe, is as the baſe and the perpendicular altitude of the water, whatever be the form and capacity of the veſſel.

Cor. Hence a ſmall quantity of water may be made to preſs with a force equal to a very great quantity; or to the greateſt weight of any kind.

PROP. V.

Water ſtands at the ſame horizontal height in communicating veſſels; whatever be their forms, or the quantities of water which they contain.

PROP. VI.

The preſſure againſt the baſe of a veſſel containing water, is as the area of the baſe multiplied by the altitude of the water.

PROP. VII.

The preſſure againſt the ſide of a veſſel, containing water, is as the ſquare of the altitude of the water.

PROP. VIII.

In any plane that ſuſtains water, the centre of preſſure is the ſame as the centre of oſcillation, ſuppoſing the axis of motion to be at the interſection of the plane with the ſurface of the fluid.

Cor. From theſe propoſitions it is eaſy to ſee what ought to be the ſtrength of the bottoms, and ſides of veſſels containing common water, a cubic foot of which weighs about 76 ℔ Troy: and what is the beſt method of making dams and ſluices for reſervoirs.

SECT. III.

OF BODIES IMMERSED IN WATER.

PROP. I.

The preſſure upon a ſolid body immerſed in water, increaſes in proportion to the height of the water above the ſolid; and if the body is ſmall, the difference of the preſſure upon its upper and under ſurfaces will approach to equality, as the height is increaſed.

Cor. Hence brittle ſhells, &c. are not cruſhed tho at the bottom of the ocean: a bottle immerſed will break in pieces, or continue entire, according as it is empty, or full; and according to the preſſure requiſite to drive in the cork: and hence water may be taken up unmixed, from different depths in the ocean.

PROP. II.

If a body of the ſame ſpecific gravity with water; that is, of the ſame weight when their bulks are equal; is immerſed in water, it will remain wherever it is placed: if it is of greater ſpecific gravity than water, it will ſink to the bottom: and if it is of leſs ſpecific gravity, it will riſe to the top and ſwim.

Cor. A hollow body, ſpecifically lighter than water, may have its ſpecific gravity increaſed by putting into it, before its immerſion, either water, or any body that is ſpecifically heavier than water. And a hollow body, ſpecifically lighter than water, and containing air, may be made to take different altitudes, after its immerſion, according as more or leſs water is forced into it by preſſure upon the ſurface of the water.

PROP. III.

A body ſpecifically heavier than water, and ſuſpended in it, loſes as much of its weight as a quantity of water weighs that is equal in bulk to the body ſuſpended.

Cor. The water gains as much weight as the body loſes; and if a body ſpecifically lighter than water, is held in it by force, it will add to the weight of the water as much as the difference between the ſpecific gravity of the body and of the water, and as much as a body of the ſame bulk and ſpecifically heavier would do, if ſuſpended in it.

PROP. IV.

When a body is ſpecifically heavier than water, it will ſink in it with a force that is equal to the exceſs of its own weight, above the weight of an equal bulk of the water.

PROP. V.

If a body is ſpecifically lighter than water, it will riſe in it with a force that is equal to the exceſs of an equal bulk of water above the weight of the body.

PROP. VI.

A quantity of water equal in bulk to the immerſed part of a body ſwimming on its top, is equal in weight to the whole body.

Cor. 1. Hence the different weights put into a veſſel floating in water, may be known by the different depths to which the veſſel ſinks; and the whole veſſel with its contents, is to the part of it immerſed, as the ſpecific gravity of the water is to that of the veſſel with its contents.

Cor. 2. A ſhip that is ſunk may be raiſed from the bottom, and brought to the ſhore, if connected by a tight cable, with a light veſſel that floats above her, during a neap tide; but if ſhe is ſunk where there are ſmall or no tides, by pumping a great quantity of water into the light veſſel before the ſhip is connected with it, and then pumping the water out of the veſſel.

PROP. VII.

The heavieſt ſolid may be made to ſwim in water, by keeping the water from preſſing upon its upper ſurface; and the lighteſt ſolid may be made to remain at the bottom, by keeping the water from preſſing upon its under ſurface.

SECT. IV.

METHODS OF FINDING THE SPECIFIC GRAVITY OF BODIES.

PROP. I.

If bodies which are compared together are of equal bulks, their abſolute weights will be as their ſpecific gravities: if they are of the ſame ſpecific gravity, their abſolute weights will be as their bulks: and if their abſolute weights are equal, their bulks will be reciprocally as their ſpecific gravities.

PROP. II.

If bodies which are compared together, are not equal in their bulks, ſpecific gravities, or abſolute weights, their abſolute weights will be in a compound ratio of their ſpecific gravities and bulks: their ſpecific gravities will be in a compound ratio of their abſolute weights directly, and bulks inverſely: and their bulks will be in a compound ratio of their abſolute weights directly, and their ſpecific gravities inverſely.

Cor. The abſolute weight of any body is as the product got by multiplying its bulk by its ſpecific gravity. The ſpecific gravity of any body, is as the quotient of its abſolute weight, divided by its bulk. And the bulk of any body, is as the quotient of its abſolute weight, divided by its ſpecific gravity.

Of a ſtandard fluid, by means of which the ſpecific gravity of all bodies, both fluid and ſolid, may be compared.

PROP. III.

Pure rain-water is the beſt ſtandard fluid; and, when it is uſed as ſuch, its temperature, and that of the body examined, ſhould be carefully marked; otherwiſe errors muſt take place in making tables of the ſpecific gravities of bodies.

PROP. IV.

The ſpecific gravity of fluids may be found by the following methods.

By weighing equal bulks of the fluids, meaſured in the ſame veſſel; and their ſpecific gravities will be as their weights.

By immerſing in rain-water a ſtraight glaſs tube, open at both ends, containing the fluid to be examined, if it does not mix with water, and is ſupported by the preſſure of the rain-water; ſo that the height of the rain-water, and of the fluid in the tube, will be reciprocally as their denſities.

By an incurvated glaſs tube, into one leg of which the ſtandard fluid is put, and into the other, the fluid examined, with quickſilver interpoſed in ſuch a manner, as that the quickſilver will riſe to an equal height in both legs; and then the ſpecific gravity of the ſtandard fluid will be to that of the fluid examined, inverſely as the heights of their upper ſurfaces above the quickſilver.

By the hydrometer, which when put into a fluid of greater ſpecific gravity than the ſtandard fluid, will riſe above the ſtandard mark; and when put into a fluid of leſs ſpecific gravity, will ſink below the ſtandard mark; ſo that the denſities of the fluids compared, will be inverſely as the parts immerſed.

By a ſeries of glaſs bubbles, increaſing and decreaſing in ſpecific gravity, in a certain ratio, from the ſtandard fluid, ſo that when they are immerſed in a fluid, the ſpecific gravity of which is ſought, that one which is of the ſame ſpecific gravity with the fluid, will remain wherever it is placed.

By ſuſpending in the fluids examined, at the arm of a hydroſtatic balance, a ſolid body of greater ſpecific gravity than the fluids, and putting weights to make an equilibrium, into the one or the other ſcale, according as the ſolid body riſes or ſinks; and theſe weights will give the ſpecific gravity of the fluids examined, if the weight of the ſolid body in air, and in water, is known.

Of unavoidable inaccuracies which take place in practiſing the firſt four of theſe methods.

PROP. V.

The ſpecific gravity of a ſolid, that is denſer than rain-water, may be found by the following methods.

By ſuſpending the body in rain-water, at the arm of a balance, and meaſuring the force with which it deſcends; which will give its denſity compared to that of the water.

By this method. Weigh it firſt in air, then in the ſtandard fluid by the hydroſtatic balance; make the difference of the two weights a diviſor, the weight of the ſolid in the air a dividend; and the quotient will be the number ſought: that is, if the ſtandard fluid is taken as unity, the quotient will have the ſame proportion to it, which the ſpecific gravity of the ſolid has to that of the fluid.

PROP. VI.

The ſpecific gravity of a ſolid, not ſo denſe as water, may be found by the following methods.

By immerſing the body, and an inverted balance to which it is tied, in the ſtandard fluid; and meaſuring by another balance the force with which the body aſcends, and that will give its ſpecific gravity.

By this method. Weigh in air, and in ſtandard water, a piece of metal that is ſufficient to make the body ſink in water; tie them together with a horſe-hair; weigh them, when united, firſt in air, and then in rain-water; ſubtract the weight of the metal when alone in the water, from its weight in the air; and the remainder will be the weight of as much water as is equal in bulk to the metal. Subtract the weight of the united bodies in the water, from their weight in the air, and the remainder will be the weight of as much water, as is equal in bulk to the united bodies. Take the weight of as much water as has been found to be equal in bulk to the metal, from the weight of as much water as is equal in bulk to the united bodies; and the remainder will be the weight of as much water as is equal in bulk to the lighter body; and the proportion which this weight bears to the weight of the lighter body, in air, will be the proportion of the ſpecific gravity of the water, to the ſpecific gravity of the lighter body.

This operation will not be neceſſary if there is a contrivance in the hydroſtatic bucket, for keeping bodies immerſed that are ſpecifically lighter than rain-water.

PROP. VII.

If the ſpecific gravity of two metals is known, and if a compound, of any ſhape whatever, is made from them by fuſion, the proportion of the two metals in the compound may be thus diſcovered. Find the bulk of the compound, and the bulk of an equal weight of each of the two metals; and then as the difference of the ſpecific gravity of the compound, and the lighter metal, is to the difference of the ſpecific gravity of the compound, and the heavier metal; ſo is the bulk of the heavier metal to the bulk of the lighter metal; and ſo, conſequently, is the proportion of their parts in the compound.

This method is only exact when the two bodies in the compound are united by appoſition, without any diminution or increaſe of their bulks. For ſome bodies, it would ſeem, penetrate each other when mixed; the ſolid parts of the one entering the pores of the other. Thus, if gold is mixed with biſmuth, zinc, or lead: ſilver with tin, biſmuth, zinc, or quickſilver: copper with tin, iron, or zinc: lead with biſmuth, mercury, or zinc: and wax with ox-tallow; the bulk of the compound is leſs than it ought to be by the addition of the bulk of the one body, to that of the other. On the other hand, the pores of bodies are ſometimes enlarged by mixture. Thus, iron mixed with tin, zinc, biſmuth, regulus of antimony, or lead; and tin with zinc, biſmuth, lead, or regulus of antimony; and mercury with tin, biſmuth, zinc, or lead; are increaſed in bulk in a greater proportion than ought to ariſe from their bulks when added.

PROP. IX.

If we would find out the ſpecific gravity of a body that is ſoluble in the ſtandard fluid, it may firſt be dipt in melted bees wax to prevent its ſolution; and as white bees wax may be refined ſo as to become of the ſame ſpecific gravity with water, there will be no error in the calculation. Or it may be weighed by the hydroſtatic balance in a fluid of known ſpecific gravity, that will not diſſolve it; and its denſity compared to that fluid being known, its ſpecific gravity compared to the ſtandard fluid will be known likewiſe.

PROP. X.

When the ſpecific gravity of a body is examined, care muſt be taken that it does not imbibe water: that the air is expelled from all its cavities: and that there is no repulſion between the water and it; for even gold will not ſink, when immerſed in water, if it is beaten thin, and if the corpuſcular repulſion between it and water is not overcome.

PROP. XI.

If great accuracy is required in aſcertaining the ſpecific gravity of a body, it is neceſſary that the denſity of the atmoſphere ſhould be known; for according as it varies, ſo will the weight of the body that is weighed. Thus, when the atmoſphere is of a certain denſity, if a piece of a gold is ſuſtained by a braſs weight, in the oppoſite ſcale of a balance; and the firſt has double the ſpecific gravity of the ſecond; if they are afterwards weighed when the atmoſphere is lighter, the gold will loſe but one half of what the braſs will loſe; and, conſequently, the former equilibrium will be deſtroyed.

Cor. Hence, a buyer of gold will gain by a lighter atmoſphere, and loſe by a heavier: but the reverſe will take place, if what he buys is ſpecifically lighter than the weights that are uſed.

A Table of the ſpecific gravity of ſome bodies, when they and the ſtandard fluid are at the temperature 50°; when the barometer is at its mean height; and the balance turns with the fiftieth part of a grain.

Rain-water	1,000
Diſtilled water	993
Salt water from the mouth of a river	1,030
Salt water from the ocean	1,680
Aqua fortis	1,300
Aqua regia	1,244
Oil of linſeed	942
Oil of ſaſſafraſs	1,099
Alcohol	742

Spirit of wine	876
Oporto wine	993
Burgundy wine	964
Canary wine	1,043
Diſtilled vinegar	1,011
Cow's milk	1,040
Goat's milk	1,009
Quickſilver diſtilled	14,000
Quickſilver from a ſhop	13,500
Sal gem	2,143
Alum	1,714
Nitre	1,900
Amber	1,050
Sulphur	1,800
Diamond	3,400
Agate	2,512
Rock chryſtal	2,659
Iceland chryſtal	2,720
Aſbeſtos	2,943
Granite ſtone	2,460
Chalk	2,470
Talc	2,654
Loadſtone	1,840
Fine gold	19,640
Britiſh ſtandard gold	18,888
Platina as from the mine	16,995
Parts of platina ſelected	27,500
Platina cupelled with lead	19,240
Fine ſilver	11,091
Britiſh ſtandard ſilver	10,535
Lead when pure	11,325
Copper	9,000

Tin	7,400
Iron	8,000
Steel	7,704
Oak	925
Fir	546
Lignum vitae	1,367
Lignum guiacum	1,337
Cork	240
Bees wax refined	1,000

SECT. V.

OF THE MOTION OF WATER BY ITS OWN PRESSURE.

PROP. I.

If a veſſel contains water, and an aperture is made in its baſe, the water will flow with degrees of velocity conſtantly diminiſhing, and the velocities, at given times, will be in the duplicate ratio of the altitudes of the water above the aperture; provided no ſenſible retardation is produced by the corpuſcular attraction, the friction, the reſiſtance of the air, or the reſervoir's being ſo ſmall, compared to the aperture, that there is an irregular communication of the preſſure.

PROP. II.

If an aperture is made in the ſide of a veſſel, the water will flow till its ſurface is in the ſame level with the loweſt part of the aperture; and its velocity will be as expreſſed in the laſt propoſition.

SECT. VI.

OF THE QUANTITY OF WATER THAT FLOWS THRO APERTURES IN VESSELS.

PROP. I.

When equal apertures are made in a veſſel, the quantities of water that flow in a given time will be as the ſquare roots of the heights of the water above the apertures; provided no ſenſible retardation is produced by the cauſes above mentioned.

PROP. II.

The height of water in a veſſel being the ſame, and the apertures different, the quantity that flows in a given time, will be as the apertures; and if they are circles, in the duplicate ratio of the diameters.

Cor. The quantities which flow are in a ratio compounded of the times, of the apertures, and of the ſquare roots of the heights.

PROP. III.

If cylindric veſſels containing water have the ſame heights, and the ſame diameters, but unequal apertures, the times in which they will be emptied will be to each other inverſely as the apertures. And if they have the ſame heights, and unequal diameters, they will be emptied thro equal apertures, in times that are to each other as the diameters.

Cor. 1. The times in which cylindric veſſels are emptied are in a ratio compounded of the baſes, of the ſquare roots of the heights, and of the inverſe ratio of the apertures.

Cor. 2. If apertures are made at heights from the baſe of the veſſel, which heights are as the ſquares of numbers in arithmetical progreſſion, each of the diviſions of the veſſel between the apertures will be emptied in the ſame time.

PROP. V.

If a tube is added to the apertures in the baſe of a veſſel, a greater quantity of water will flow than when there was no tube; and if the lower end of the tube is wider than the upper end, a greater quantity will flow than if the tube were a cylinder.

A Table of the quantity of water which flows in an hour, thro an aperture of an inch ſquare, when at different depths below the ſurface of the reſervoir: and of the quantity which flows in an hour, thro an aperture in the form of a parallelogram, one inch in breadth, and of the length marked in the laſt column; the upper end of the parallelogram being in the ſame plane with the ſurface of the reſervoir.

Quantity from the ſquare aperture.	Quantity from the ſlit aperture.	Depth below the ſurface.
Tons.	Tons.	Inches.
1,04	1,04	1
1,46	2,50	2
1,79	4,29	3
2,08	6,37	4
2,31	8,68	5

Quantity from the ſquare aperture.	Quantity from the ſlit aperture.	Diſtance below the ſurface.
Tons.	Tons.	Inches.
2,53	11,21	6
2,74	13,95	7
2,92	16,87	8
3,12	19,19	9
3,28	22,47	10
3,44	25,81	11
3,58	29,39	12
3,74	33,13	13
3,88	36,91	14
3,95	40,86	15
4,16	45,02	16
4,28	49,30	17
4,40	53,70	18
4,52	58,22	19
4,62	62,84	20
4,76	67,60	21
4,87	72,47	22
4,94	77,41	23
5,06	82,47	24
5,02	87,67	25

SECT. VII.

OF SPOUTING WATER.

PROP. I.

A vertical jet never riſes to an equal height with the ſurface of its reſervoir, of which theſe are the cauſes.

When the higheſt particles of the water ceaſe to have motion, they become a load, for ſome time, upon the top of the jet.

A diminution of velocity is communicated to every part of the jet, by the friction of the particles of the water, that are in contact with the aperture.

The momentum of the jet is reſiſted by the air which ſurrounds it.

Cor. Hence the jet will increaſe in thickneſs from the aperture to its ſummit.

PROP. II.

The firſt of theſe cauſes may be removed by giving an inclination to the jet; but then it is not truly vertical. The ſecond cannot be entirely removed: it may, however, be diminiſhed by a proper conduit-pipe, and a thin plate of metal inſtead of a conical ajutage, or ſpouting pipe. The third may be diminiſhed by rarifying the ſurrounding air; and it may be almoſt entirely removed, if the jet is ſo ſmall that it can be placed in vacuo.

PROP. III.

There is a maximum and a minimum in diminiſhing the above cauſes of retardation. By increaſing the diameter of the aperture, the friction will increaſe only as the diameters, while the momentum will increaſe as the ſquares of the diameters; but as ſoon as the momentum deſtroys the cohesion of the water, it ceaſes to be a jet. On the other hand, when the momentum is ſo ſmall that the jet cannot act with a certain force againſt the cauſes of retardation, the jet will be broken into ſmall ſtreams, or its altitude will be diminiſhed. In order, therefore, to get the greateſt height, and greateſt ſize of a clear, unbroken column of water, the mean of the greateſt advantages muſt be taken; and this is found, by experiment, to be as in the following table; which contains the height to which jets will riſe in feet and decimal parts, from reſervoirs of different altitudes, from five feet to an hundred.

Reſ.	Jet.	Reſ.	Jet.	Reſ.	Jet.
5	4,91	20	18,82	35	31,63
6	5,88	21	19,70	36	32,47
7	6,84	22	20,58	37	33,29
8	7,80	23	21,46	38	34,11
9	8,74	24	22,33	39	34,93
10	9,68	25	23,20	40	35,74
11	10,62	26	24,06	41	36,55
12	11,55	27	24,92	42	37,35
13	12,48	28	25,78	43	38,14
14	13,40	29	26,63	44	38,93
15	14,31	30	27,48	45	39,75
16	15,22	31	28,32	46	40,53
17	16,13	32	29,16	47	41,31
18	17,03	33	30,	48	42,09
19	17,93	34	30,83	49	42,87

Reſ.	Jet.	Reſ.	Jet.	Reſ.	Jet.
50	43,65	67	56,39	84	68,40
51	44,42	68	57,12	85	69,08
52	45,19	69	57,84	86	69,76
53	45,96	70	58,56	87	70,47
54	46,72	71	59,28	88	71,14
55	47,48	72	60,	89	71,81
56	48,24	73	60,71	90	72,48
57	48,99	74	61,42	91	73,15
58	49,74	75	62,13	92	73,82
59	50,49	76	62,84	93	74,49
60	51,24	77	63,54	94	75,16
61	51,99	78	64,24	95	75,83
62	52,73	79	64,94	96	76,49
63	53,47	80	65,64	97	77,15
64	54,20	81	66,33	98	77,81
65	54,93	82	67,02	99	78,47
66	55,66	83	67,71	100	79,12

It is found, by experiments, that if a reſervoir is five feet high, and its conduit-pipe one inch and three fourths in diameter, and not exceeding about 100 feet in length, it will admit of an ajutage, from one fourth to three eights of an inch, in diameter; and ſo on as follows.

Height of the reſervoir.	Diameter of the conduit-pipe.	Diameter of the ajutage.
Feet.	Inches.	Inches.
5	$1\frac{3}{4}$	$\frac{1}{4}$ to $\frac{3}{8}$
10	2	$\frac{1}{4}$ to $\frac{1}{2}$
15	$2\frac{1}{4}$	$\frac{1}{2}$

Height of the reſervoir.	Diameter of the conduit-pipe.	Diameter of the ajutage.
Feet.	Inches.	Inches.
20	$2\frac{1}{2}$	$\frac{1}{2}$
25	$2\frac{3}{4}$	$\frac{1}{2}$
30	3 to $3\frac{1}{2}$	$\frac{1}{2}$ to $\frac{3}{4}$
40	$4\frac{1}{4}$	$\frac{3}{4}$
50	5	$\frac{3}{4}$
60	$5\frac{3}{4}$ to 6	1
80	$6\frac{1}{2}$ to 7	$1\frac{1}{4}$
100	7 to 8	$1\frac{1}{4}$ to $1\frac{1}{2}$

PROP. IV.

Oblique and horizontal jets are not ſubject to the firſt of the cauſes which retard the vertical jets. Their momentums and amplitudes may be aſcertained by the above propoſitions, and by thoſe concerning projectiles.

PROP. V.

If a ſemicircle is drawn upon the altitude of a reſervoir as a diameter; and if a perpendicular is drawn from any point in that ſemicircle to the diameter, twice the length of the perpendicular will be the amplitude of a horizontal jet, iſſuing from an aperture at the point where the perpendicular meets the reſervoir; the amplitude being meaſured upon a horizontal plane paſſing thro the baſe of the reſervoir. If the amplitudes are not meaſured upon this plane, but upon one parallel to it, and below the baſe of the reſervoir; the amplitudes will be different, as determined concerning bodies projected from points that are above the plane which they ſtrike.

Cor. The greateſt amplitude meaſured upon a plane paſſing thro the baſe of the reſervoir, will be got by an aperture made in the centre of the ſemicircle; and if one aperture is made at the ſame diſtance from the baſe, that another is from the ſurface, of the reſervoir; their amplitudes will be equal.

SECT. VIII.

OF CONDUCTING WATER IN PIPES.

PROP. I.

Water will riſe to the ſame horizontal level in conduit-pipes; but will not run unleſs the place of delivery is lower than the reſervoir: and the velocity with which it will run at different diſtances below the height of the reſervoir, will be in the ſubduplicate ratio of the diſtances.

PROP. II.

Conduit-pipes will deliver but a ſmall quantity of water, unleſs they are freed of the air which is continually entering with the water; and this is done by cocks, and riders of different kinds.

Of bringing water from reſervoirs to towns and houſes. Of the ſtrength to be given to conduit-pipes in different parts of their courſe: and of the beſt method of laying them; according to the places over which they are carried, and the countries where they are uſed.

Of ſtoppers to pipes, moved by the water; and of the beſt kinds of reſervoirs.

Of the friction, and momentum of water, in pipes; when moved by its own gravity, and by the force of machines.

SECT. IX.

OF SPRINGS, OR NATURAL FOUNTAINS.

PROP. I.

It has been found, by experiment, that the fountains in ſome countries, yield more water than what comes from the clouds, in thoſe countries, in the form of rain and ſnow.

Of the method of meaſuring the water that is yielded by fountains; and of pluviometers of different kinds.

Of the hypotheſes that have been formed to account for the produce of fountains, by ſubterranean ſupplies from the ocean.

PROP. II.

The water produced from the atmoſphere, by rain, ſnow, ſleet, &c. and by condenſed vapours, has been found to afford a ſufficient ſupply for the fountains, in particular diſtricts, which receive no additions from diſtant ſubterraneous lakes, or brooks.

Of common fountains; which are temporary, or perennial: of intermitting, and reciprocating fountains: and the cauſes of them.

Of pit-wells; and the beſt method of making, and preſerving them in perfection.

BIBLIOTHECA BODLEIANA

SECT. X.

OF RIVERS.

OF REGULAR RIVERS.

PROP. I.

In a regular river, the particles of the water are equably accelerated as they come from the reſervoir or ſource; and in the ratio of the ſquares of perpendiculars drawn from the river, to a horizontal plane paſſing thro the ſurface of the reſervoir.

Cor. Hence the lower part of the water will move faſter than the upper; and the celerity of the lower and upper parts will approach to equality, as the river goes on.

PROP. II.

The water of a regular river diminiſhes in depth as it recedes from the ſource; but while it is in a permanent ſtate, the ſame quantity of water flows in the ſame time, thro every ſection of it.

NB. The Remainder of this Volume – had never
been printed – when this was obtained – Sept.r 1782.